# PHYS 301
# Electricity and Magnetism

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Today!

- Quiz!!
- The Gradient
- Intro to spherical coordinates
- The Divergence
- The Curl

# Differential Calculus

- For a function of one variable, *f(x)*,

  *df/dx* is the **slope** of the curve *f(x)*

- For a scalar function of two, three, or more variables [*e.g.*, g(x,y), h(x,y,z), *etc*.], the "**slope**" (how fast the function varies) depends upon the **direction** one moves
- The **GRADIENT** of the function serves as the generalization of the 1D derivative:

$$\vec{\nabla}P(x,y,z) \equiv \hat{i}\,\frac{\partial P}{\partial x} + \hat{j}\,\frac{\partial P}{\partial y} + \hat{k}\,\frac{\partial P}{\partial z}$$

in Cartesian coords only!

# Differential Calculus: The Gradient

- The del operator is defined as

$$\vec{\nabla} \equiv \hat{i}\,\frac{\partial}{\partial x} + \hat{j}\,\frac{\partial}{\partial y} + \hat{k}\,\frac{\partial}{\partial z}$$

Know this!

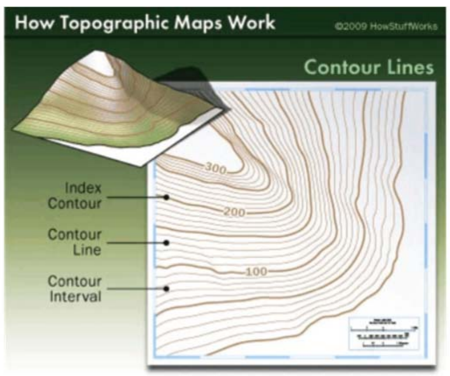
- Del acting on a scalar is called the **gradient**

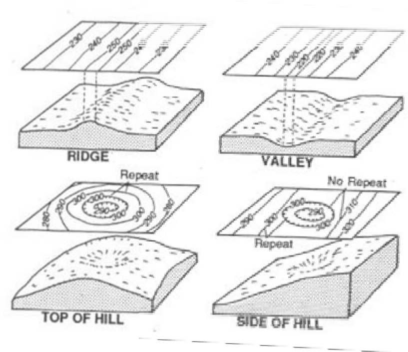
$$\vec{\nabla}P(x,y,z) \equiv \hat{i}\,\frac{\partial P}{\partial x} + \hat{j}\,\frac{\partial P}{\partial y} + \hat{k}\,\frac{\partial P}{\partial z}$$

# Visualization: Contour Lines

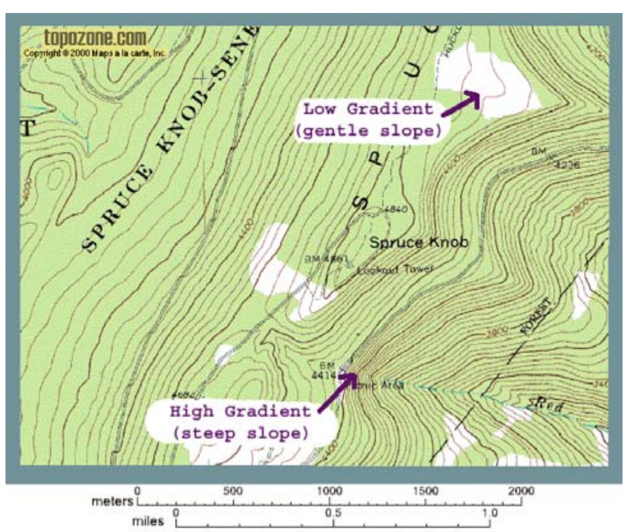
Consider a function of two variables:

h(x,y) = height above sea level




# Contour Lines

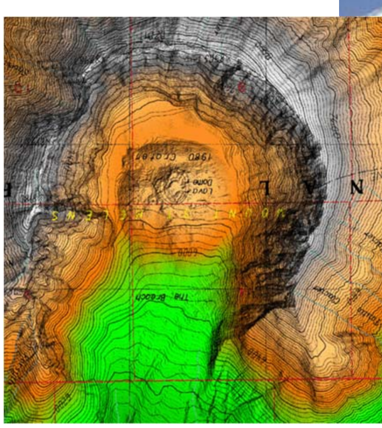


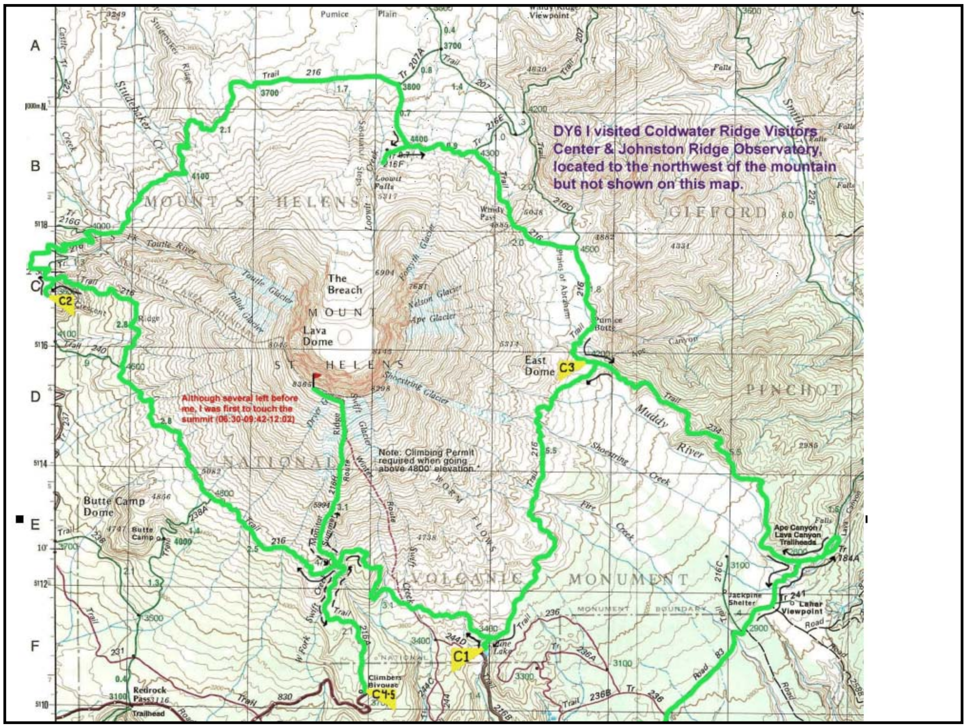
# Slope is related to the Gradient

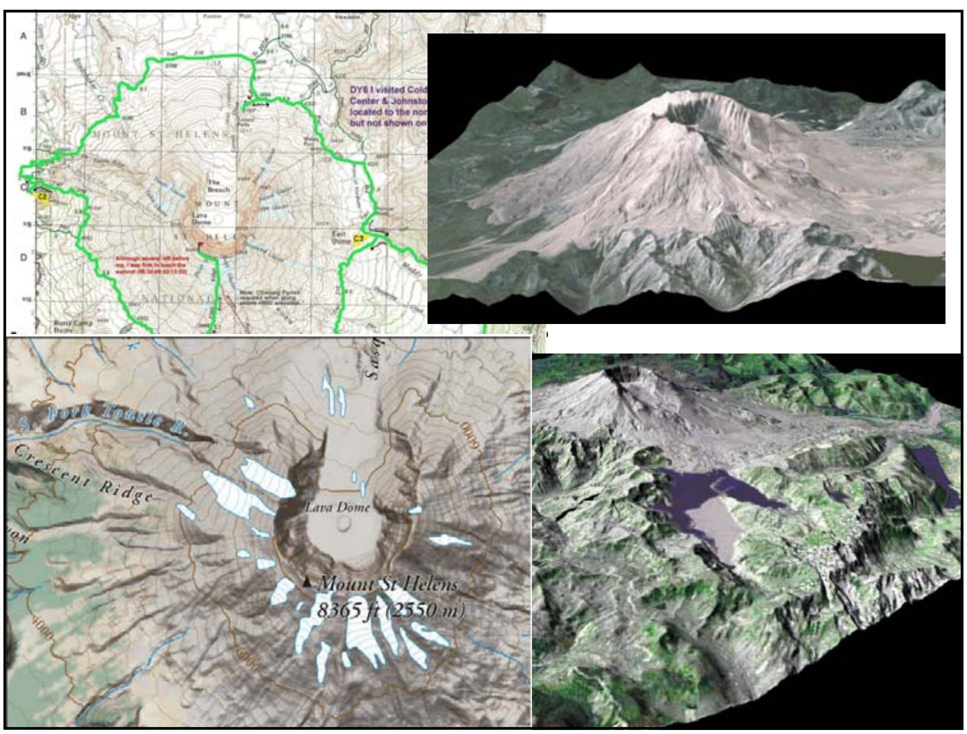
Let V(x,y) represent the elevation (height) as a function of position (lat/long)

Here, the arrows represent $-\vec{\nabla} V(x,y)$


# Mt. St. Helens


DY6 I visited Coldwater Ridge Visitors Center & Johnston Ridge Observatory, located to the northwest of the mountain but not shown on this map.
Although several left before me, I was first to touch the summit (06:30-09:42-12:02)
Note: Climbing Permit required when going above 4800' elevation.
The Breach
Lava Dome
East Dome
Butte Camp Dome
Pumice Butte
Muddy River
Ape Canyon / Lava Canyon Trailheads
Lahar Viewpoint
Jackpine Shelter
Redrock Pass Trailhead
Climbers Bivouac
C1
C2
C3
PINCHOT
GIFFORD
VOLCANIC MONUMENT


S. Fork Toutle R.
Crescent Ridge
Lava Dome
Mount St Helens
8365 ft (2550 m)

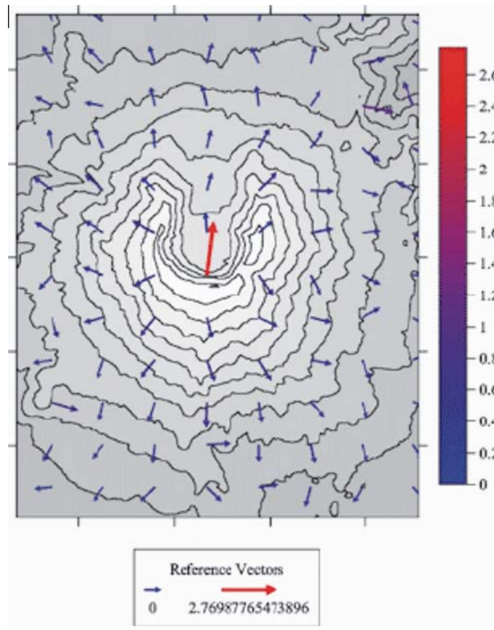
## The Gradient

$$\vec{\nabla} V(x,y)$$

Indicates direction of greatest increase of elevation

$$-\vec{\nabla} V(x,y)$$

Indicates direction of greatest decrease of elevation




## In summary: The Gradient

- Del acting on a scalar is called the **gradient**

$$\vec{\nabla} P(x,y,z) \equiv \hat{i}\frac{\partial P}{\partial x} + \hat{j}\frac{\partial P}{\partial y} + \hat{k}\frac{\partial P}{\partial z}$$

- $\vec{\nabla} P(x,y,z)$ points in the direction of maximum increase in the function $P(x,y,z)$

- $\left|\vec{\nabla} P(x,y,z)\right|$ gives the "slope" (rate of increase) along this maximal direction